

Suffolk County Community College
Michael J. Grant Campus
Department of Mathematics

Wednesday, May 13, 2026

MAT 101
A Survey of Mathematical Reasoning
Final Exam

Instructor:

Name: Alexander Kasiukov
Office: Suffolk Federal Credit Union Arena, Room A-109
Phone: (631) 851-6484
Email: kasiuka@sunysuffolk.edu
Web Site: <http://kasiukov.com>

Please print the requested information in the spaces provided:

Student:

Name:

Student Id:

Email:

include to receive the final grade via email ONLY if you are not getting email updates

- *Any violation of academic integrity on this exam will result in a failing grade for the whole course.*
- *Notes and books are permitted, but cannot be shared.*
- *Graphing calculators, smartwatches, computers, cell phones and any other communication-capable devices are prohibited. Their mere presence in the open — even without use — is a violation of academic integrity.*
- *You will not receive full credit if there is no work shown, even if you have the right answer. Please don't attach additional pieces of paper: if you run out of space, please ask for another blank final.*

Existential presupposition is in effect for *(some x are y)* statements. The proposition *(some cats are smart)* means that the universe of discourse contains at least one cat which is smart. Existential presupposition is not in effect for *(all x are y)* statements. The proposition *(all unicorns are mammals)* means that every unicorn existing in the universe of discourse must be a mammal, but it does not assume that even a single unicorn exists.

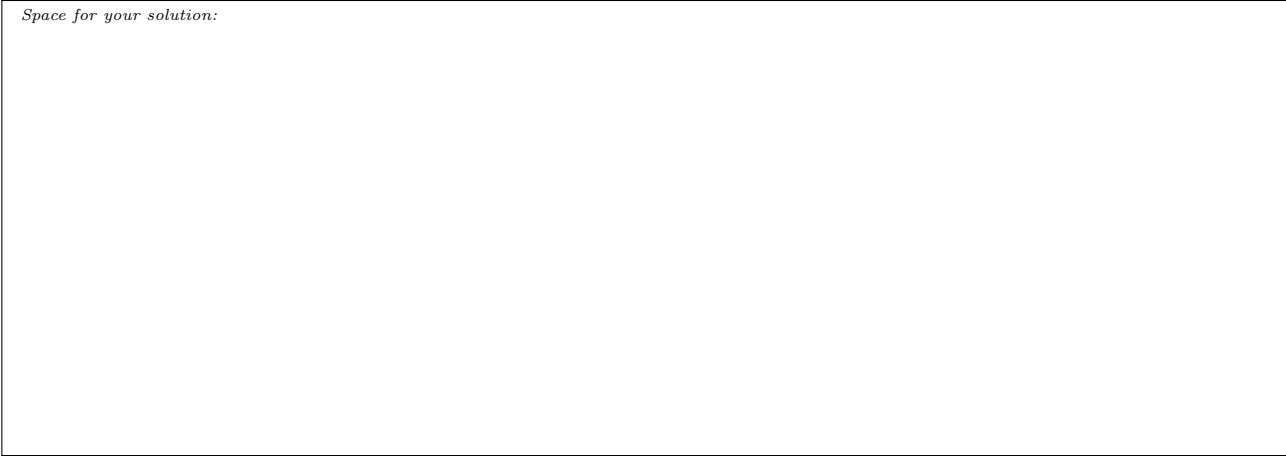
Problem 1. Consider the following syllogism:

Some cats are mammals.
Some mammals are animals.

Some cats are animals.

(1). Draw a Venn diagram showing the categories of this syllogism.

Space for your solution:



(2). Express the assumptions of this syllogism graphically in the Venn diagram.

Space for your solution:



(3). Give a reason why this syllogism is a valid argument, or provide a counterexample showing that it is invalid.

Space for your solution:

(4). Using predicates and quantifiers, express this syllogism in the form of a Gentzen-style argument of predicate logic.

Space for your solution:

Problem 2. Suppose three suspects were caught in an art museum. Before they surrendered to police, they agreed that each one of them will tell a half-truth to their interrogators. They were separated and asked two questions: which painting they wanted to steal and who commissioned them.

- The first said that they wanted to steal the Rembrandt and the crime boss Big Joe promised to buy it.
- The second said that they came to get the Degas and stated that it was definitely not the Big Joe who sent them.
- The last suspect claimed that they came to steal the Monet, and were commissioned by the notorious Lucky Sam.

(1). Using disjunction to represent a half-true statement, we can write the information conveyed by the first suspect as

$$\text{Rembrandt} \vee \text{Joe.}$$

Assuming, in addition to the information gained in the interrogations, that the Rembrandt was not the target of the theft, formulate a valid argument in Gentzen’s notation:

$$\frac{\text{Rembrandt} \vee \text{Joe} \quad \neg\text{Rembrandt}}{\text{???}}$$

stating who commissioned the theft. For this problem, only the statement of the argument, rather than verification of its validity, is needed.

Space for your solution:

(2). Write the argument which you gave as the answer to the previous problem as a single statement of propositional logic. (Use any necessary logical connectors that may be implicit in Gentzen’s notation.)

Space for your solution:

(3). Use the truth tables to verify the validity of this argument.

Space for your solution:

(4). Suppose we record the information gained in the interrogation in the form of the following Gentzen-style argument:

$$\frac{\begin{array}{l} \text{Rembrandt} \vee \text{Joe} \\ \text{Degas} \vee (\neg \text{Joe}) \\ \text{Monet} \vee \text{Sam} \end{array}}{\text{???}}$$

Assume that in addition to this information the police knows that the thieves could only steal one painting, and that Big Joe and Lucky Sam are enemies, so the the thieves could only serve one of them but not both. How can that additional information be added to the above assumptions?

Space for your solution:

(5). **EXTRA CREDIT** Use the distributivity of conjunction with respect to disjunction:

$$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C),$$

to extract the truth from the information depicted in the previous subproblem. Hint: you may find it convenient to write conjunction as multiplication, and disjunction as addition, so that the above information gets recorded as

$$\begin{array}{r} \text{Rembrandt} + \text{Joe} \\ \text{Degas} + (\neg \text{Joe}) \\ \text{Monet} + \text{Sam} \\ \hline \dots \\ ??? \end{array}$$

It may also be helpful to write the falsehood of a statement X as $X = 0$, and its truth as $X = 1$.

Space for your solution:

Problem 3. Consider the following argument: *Iron is gradually sinking from the mantle of earth into its core. Since iron is heavier than the rocks it displaces, this sinking decreases earth's moment of inertia. Decreasing the moment of inertia of any closed system increases its speed of rotation*¹. *If the speed of earth's rotation were to increase, days would get shorter. In fact, days are getting longer, not shorter. Thus, the earth is not a closed system.*

(1). Identify and properly name² any single gate used (explicitly or implicitly) in this argument.

Space for your solution:

(2). Identify any single predicate used in this argument.

Space for your solution:

¹This follows from the law of physics, called conservation of angular momentum. The law states that the angular momentum of a closed system remains constant over time. Since the angular momentum is the product of the moment of inertia and of the rotational speed, decrease in one of those two parameters (of a given closed system) will result in the proportional increase in another.

²i.e. as a “negation”, “conjunction”, “disjunction”, “implication”, or “equivalence”

(3). Translate this argument into the format of predicate logic ³.

Space for your solution:

³in Gentzen's notation

(4). Prove the validity of this argument.

Space for your solution:

