Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

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MAT 103
Statistics I

Final Exam: Solutions and Answers

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Problem 1. University of California, Berkeley graduate division admitted 44% of male and 35% of female applicants in the fall of 1973. Noticing this apparent discrepancy, the associate dean of the graduate school[1] asked Peter Bickel, then a professor of statistics at Berkeley, to analyze the data. The results of that analysis [2] became one of the most widely cited examples of the statistical phenomenon called *Simpson's Paradox*.

In this problem, we explore this phenomenon and its ramifications. The original paper by Bickel at al. does not contain raw data on the individual departments, but the Data Science Discovery platform [3] has a data set covering all the 12,763 applicants from the original study. It obscures specific department names, but provides obfuscated department codes (A, B, C, D, E and F) for the six most popular departments. In this problem, we will focus only on those six departments, and — in the interest of time — we will further group them into two groups: departments A-B and departments C-D-E-F. The effect of Simpson's paradox becomes only more pronounced when only these six departments are considered.

(1). Based on the aggregated six-department data:

	Male	**Female**
Accepted	1,511	557
Rejected	1,493	1,278

compute and compare the conditional probabilities:

$$P(\text{Accepted}|\text{Male}) =$$

$$P(\text{Accepted}|\text{Female}) =$$

and determine if there has been a bias against women in graduate admissions.

Space for your solution:

$$P(\text{Accepted}|\text{Male}) = \frac{1,511}{1,511 + 1,493} = \frac{1,511}{3,004} \approx 50\%$$

$$P(\text{Accepted}|\text{Female}) = \frac{557}{557 + 1,278} = \frac{557}{1,835} \approx 30\%$$

These probabilities seem to suggest a bias against women.

[1] see Cari Tuna (2009) "When Combined Data Reveal the Flaw of Averages", A Wall Street Journal interview with Peter Bickel, https://www.wsj.com/articles/SB125970744553071829

[2] Bickel, P. J., Hammel, E. A., and O'Connell, J. W. (1975) "Sex bias in graduate admissions: Data from Berkeley", Science, 187, 398–403, http://brenocon.com/science_1975_sex_bias_graduate_admissions_data_berkeley.pdf

[3] Berkeley's 1973 Graduate Admissions Dataset, Data Science Discovery, University of Illinois at Urbana-Champaign, https://discovery.cs.illinois.edu/dataset/berkeley/

(2). Graduate admission decisions are made by individual departments. In the attempt to "look for the responsible parties", Professor Bickel and his colleagues analyzed data for each of the 101 departments separately. We will use a much more coarse analysis, grouping the six most popular departments into two groups and analyzing the admissions data for those two groups.

Here is the statistics for the Easy-to-get-into Departments (designated as "A" and "B" in the Data Science Discovery dataset):

Easy	**Male**	**Female**
Accepted	1,178	106
Rejected	520	27

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and for the Hard-to-get-into Departments (designated as "C", "D", "E" and "F" in the Data Science Discovery dataset):

Hard	**Male**	**Female**
Accepted	333	451
Rejected	973	1,251

.

Compute and compare the conditional probabilities:

$$P(\text{Accepted}|\text{Male}) =$$

$$P(\text{Accepted}|\text{Female}) =$$

separately for the easy-to-get-into and hard-to-get-into departments.

Space for your solution:

For the easy-to-get-into departments:

$$P(\text{Accepted}|\text{Male}) = \frac{1,178}{1,178 + 520} = \frac{1,178}{1,698} \approx 69\%$$

$$P(\text{Accepted}|\text{Female}) = \frac{106}{106 + 27} = \frac{106}{133} \approx 80\%.$$

For the hard-to-get-into departments:

$$P(\text{Accepted}|\text{Male}) = \frac{333}{333 + 973} = \frac{333}{1,306} \approx 25\%$$

$$P(\text{Accepted}|\text{Female}) = \frac{451}{451 + 1,251} = \frac{451}{1,702} \approx 26\%.$$

(3). What overall conclusion can you draw from this analysis of admissions data? Did Berkeley discriminate against women in their fall 1973 graduate admissions?

Space for your solution:

The department type was a stronger predictor of admission than the gender of the applicant. Women were more likely to apply to the hard-to-get-into departments, while men disproportionately applied to the easy-to-get-into departments, thus the aggregation of the data from all six departments obscured the effect of the department choice on the admission outcomes, creating an illusion of a bias against women.

When the two department groups are analyzed separately, the effect of the department choice is separated from the effect of gender (and the bias *in favor of* women in the easy-to-get-into departments becomes apparent).

Department choice is the decision made by the applicant, not by the school. While it is entirely possible that women suffered from bias against them on the way leading them to their department selection, graduate admission statistics does not indicate any bias against women on the part of the school.

Problem 2. In 1994, Orenthal James Simpson was accused of murderning his ex-wife, Nicole Brown Simpson. In this problem, we will examine one of the arguments presented in his defence at his murder trial [4].

(1). Approximately 3.5 million women are battered every year in the United States. Assume that they are all battered by their partners. The total of 1,432 women were murdered by their previous batterers in the United States during 1992. Compute the conditional probability

$$P\left(\begin{array}{c}\text{American}\\\text{woman was}\\\text{murdered by her batterer}\\\text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\\text{woman was}\\\text{battered}\\\text{during 1992}\end{array}\right).$$

(It may be most convenient to round this number as a reciprocal of an integer.)

Space for your solution:

$$P\left(\begin{array}{c}\text{American}\\\text{woman was}\\\text{murdered by her batterer}\\\text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\\text{woman was}\\\text{battered}\\\text{during 1992}\end{array}\right) \approx \frac{1,432}{3,500,000} \approx \frac{1}{2,444}.$$

(2). During the O. J. Simpson's murder trial, Alan Dershowitz, who "made some appearances in court but mainly served as a member of O. J.'s defense team from afar while busy with his day job, teaching at Harvard Law School" [5] as a Professor of Law, claimed that Simpson's previous accusation of spousal abuse was not particularly relevant to the case, because only about one in 2,500 men who battered their partners went on to kill them. Is Dershowitz's numerical claim supported by the statistics mentioned above?

Space for your solution:

Given that the figure $\frac{1}{2{,}444}$ found in the previous subproblem is close enough to the $\frac{1}{2{,}500}$ cited by Alan Dershowitz, we can conclude that the Professor's numerical claim is supported by the cited statistical evidence.

[4] The statistical data in this problem are taken from William P. Skorupski, Howard Wainer (2015) "The Bayesian flip: Correcting the prosecutor's fallacy", Significance, Volume 12, Issue 4, `https://rss.onlinelibrary.wiley.com/doi/epdf/10.1111/j.1740-9713.2015.00839.x` unless noted otherwise.

[5] Natalie Finn (2024), "Absolutely 100 Percent Not Guilty": 25 Bizarre Things You Forgot About the O. J. Simpson Murder Trial `https://www.eonline.com/news/1047537/absolutely-100-percent-not-guilty-25-bizarre-things-you-forgot-about-the-o-j-simpson-murder-trial`

(3). The total of $4,936$ women were murdered in the United States in 1992. Approximately 34% of murdered women are murdered by their intimate partners [6]. Estimate

- the number of women who were murdered by their partners, and
- the number of women murdered by someone else,

in 1992.

Space for your solution:

Using the total and the percentage mentioned in this subproblem, we can estimate the total number of women murdered in 1992 by their partners as $4,936 \cdot 34\% \approx 1,678$, and by somebody other than their partner as $4,936 - 1,678 = 3,258$.

(4). In 1992, the total population of women in the United States was approximately 125 million. Using all statistical information given and computed so far, and assuming that

$$P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered by her partner}\\ \text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered by someone else}\\ \text{during 1992}\end{array}\right) = P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered by her partner}\\ \text{during 1992}\end{array}\right),$$

estimate the number of women who were battered by their partners, and murdered by someone other than their partner, in 1992.

Space for your solution:

Denoting the number in question as x and using the previous subproblem, we get:

$$P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered by her partner}\\ \text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered by someone else}\\ \text{during 1992}\end{array}\right) = \frac{x}{3,258}.$$

The data from this and the first subproblem yield: $P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered by her partner}\\ \text{during 1992}\end{array}\right) = \frac{3.5}{125}.$

The independence of events assumed in this subproblem leads to the proportion $\frac{x}{3,258} = \frac{3.5}{125}$, which can be solved for x:

$$x = \frac{3,258 \cdot 3.5}{125} \approx 91.$$

[6] See "Female Murder Victims and Victim-Offender Relationship, 2021" by the Bureau of Justice Statistics `https://bjs.ojp.gov/female-murder-victims-and-victim-offender-relationship-2021`. Note that we are assuming that this percentage in 1992 was similar to the one reported for 2021.

(5). Using all statistical data given or found so far, estimate the probability

$$P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered by her partner}\\ \text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered}\\ \text{during 1992}\end{array}\cap\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered}\\ \text{during 1992}\end{array}\right).$$

Space for your solution:

$$P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered by her partner}\\ \text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered}\\ \text{during 1992}\end{array}\cap\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered}\\ \text{during 1992}\end{array}\right)\approx\frac{1,432}{1,432+91}\approx 94\%.$$

(6). In view of all statistical data given or computed so far, do you think Professor Dershowitz gave the court and the jury a reasonable argument?

Space for your solution:

The relevance of the prior spousal abuse to the murder case is in the difference between

$$P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered by her partner}\\ \text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered}\\ \text{during 1992}\end{array}\right)\approx 34\%\quad\text{and}$$

$$P\left(\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered by her partner}\\ \text{during 1992}\end{array}\middle|\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{battered}\\ \text{during 1992}\end{array}\cap\begin{array}{c}\text{American}\\ \text{woman was}\\ \text{murdered}\\ \text{during 1992}\end{array}\right)\approx 94\%.$$

Mistaking the first for the second is called *the defence attorney's fallacy*, which Professor Dershowitz certainly committed. His numerical claim was correct, but irrelevant; his assertion of irrelevance of prior spousal abuse to the murder case was supported neither by what he himself presented to the court, nor by the additional data considered here. More statistical data is needed to settle that issue, but if the simplifying assumptions made here [a] are even remotely reasonable, Professor Dershowitz's irrelevance claim is not merely unsubstantiated, but dramatically wrong: *the information about O. J. Simpson's history of spousal abuse was very relevant to his murder case.*

[a] In (1), we assumed that all battered women are battered by their partners; in (3), we extrapolated the data from 2021 back to 1992; in (4), we assumed independence of the two events. Given their continuing interaction, we also considered O. J. Simpson as a current (rather than former) partner of the victim.

Problem 3. A grain mill manufactures 100-pound bags of flour for sale in restaurant-supply warehouses. Historically, the weights of bags of flour manufactured at the mill were normally distributed with a mean $\mu = 100$ pounds and a standard deviation $\sigma = 15$ pounds.

(1). What is the probability that the weight of a randomly selected bag of flour falls between 94 and 106 pounds? Use the table of Standard Normal Distribution included at the end of this exam.

Space for your solution:

The z-score $\frac{x-\mu}{\sigma}$ becomes $\frac{94-100}{15} = -0.4$ for $x = 94$ and $\frac{106-100}{15} = 0.4$ for $x = 106$. Using the table of standard normal distribution, we get:

$$P(94 < x < 106) = P(-0.4 < z < 0.4) = 2 \cdot P(0 < z < 0.4) = 2 \cdot 0.1554 = 0.3108.$$

(2). If samples of 36 bags are taken, what is the $\sigma_{\bar{X}}$, the standard error of the mean?

Space for your solution:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{36}} = 2.5.$$

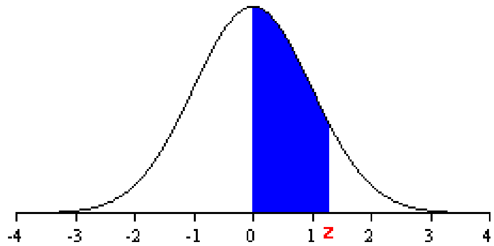
(3). What is the probability that a sample of 36 bags of flour has a mean weight between 94 and 106 pounds?

Space for your solution:

In the manner analogous to finding the z-score for the single bag of flour, the z-score for the sample $\frac{\bar{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$ becomes $\frac{94-100}{2.5} = -2.4$ for $x = 94$ and $\frac{106-100}{2.5} = 2.4$ for $x = 106$. Using the table of standard normal distribution, we get:

$$P(94 < x < 106) = P(-2.4 < z < 2.4) = 2 \cdot P(0 < z < 2.4) = 2 \cdot 0.4918 = 0.9836.$$

Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.10	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.20	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.30	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.40	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.50	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.60	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.70	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.80	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.90	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.00	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.10	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.20	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.30	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.40	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.50	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.60	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.70	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.80	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.90	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.00	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.10	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.20	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.30	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.40	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.50	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.60	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.70	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.80	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.90	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.00	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990