Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

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MAT 111: Algebra-II

Final Exam: Solutions and Answers

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Problem 1. Write the following fraction without using negative or fractional exponents. You can use only radicals, fractions and positive integer exponents in the final answer.

$$\frac{\sqrt[3]{300\ x^7y^4}}{5\sqrt[3]{9\ x^3y^6}}.$$

Space for your solution:

$$\frac{\sqrt[3]{300\ x^7y^4}}{5\sqrt[3]{9\ x^3y^6}} = \boxed{\text{write the roots as fractional exponents}} = \frac{\left(300\ x^7y^4\right)^{\frac{1}{3}}}{5\cdot(9\ x^3y^6)^{\frac{1}{3}}}$$

$$= \boxed{\text{factor 300 and 9 into prime factors}} = \frac{\left(2^2\cdot 3\cdot 5^2\ x^7y^4\right)^{\frac{1}{3}}}{5\cdot(3^2\ x^3y^6)^{\frac{1}{3}}}$$

$$= \boxed{\text{distribute and multiply the exponents}} = \frac{2^{\frac{2}{3}}\cdot 3^{\frac{1}{3}}\cdot 5^{\frac{2}{3}}\cdot x^{\frac{7}{3}}\cdot y^{\frac{4}{3}}}{5\cdot 3^{\frac{2}{3}}\cdot x^{\frac{3}{3}}\cdot y^{\frac{6}{3}}}$$

$$= \boxed{\text{combine the exponents with the same base}} = 2^{\frac{2}{3}}\cdot 3^{\frac{1}{3}-\frac{2}{3}}\cdot 5^{\frac{2}{3}-1}\cdot x^{\frac{7}{3}-\frac{3}{3}}\cdot y^{\frac{4}{3}-\frac{6}{3}}$$

$$= \boxed{\text{compute the powers}} = 2^{\frac{2}{3}}\cdot 3^{-\frac{1}{3}}\cdot 5^{-\frac{1}{3}}\cdot x^{\frac{4}{3}}\cdot y^{-\frac{2}{3}}$$

$$= \boxed{\text{break the exponents with improper fraction powers into products}} = 2^{\frac{2}{3}}\cdot 3^{-\frac{1}{3}}\cdot 5^{-\frac{1}{3}}\cdot x\cdot x^{\frac{1}{3}}\cdot y^{-\frac{2}{3}}$$

$$= \boxed{\text{go back to radicals and fractions}} = x\sqrt[3]{\frac{2^2x}{3\cdot 5y^2}} = x\sqrt[3]{\frac{4x}{15y^2}}$$

Problem 2. Find the slope-intercept equation of the line that joins the point $(-3,5)$ and $(1,-7)$.

Space for your solution:

The equation of the line given by two points is

$$\frac{y-y_{\text{old}}}{x-x_{\text{old}}} = \frac{y_{\text{new}}-y_{\text{old}}}{x_{\text{new}}-x_{\text{old}}}$$

which in our case takes the form of $\frac{y-5}{x-(-3)} = \frac{(-7)-5}{1-(-3)} \quad\Leftrightarrow\quad \frac{y-5}{x+3} = \frac{-12}{4} \quad\Leftrightarrow\quad \frac{y-5}{x+3} = -3.$ Multiplication of both sides by $x+3$ gives the desired slope-intercept equation: $y-5=-3(x+3) \quad\Leftrightarrow\quad y-5=-3x-9 \quad\Leftrightarrow\quad y=-3x-4$. Therefore the line in question has slope -3 and y-intercept -4.

Problem 3. Solve the inequality:

$$|x + 5| \leq x - 8.$$

Space for your solution:

$$|x+5| \leq x-8 \;\Leftrightarrow\; \left[\begin{array}{l} \begin{cases} x+5 \geq 0 \\ x+5 \leq x-8 \end{cases} \\ \begin{cases} x+5 < 0 \\ -(x+5) \leq x-8 \end{cases} \end{array}\right. \;\Leftrightarrow\; \left[\begin{array}{l} \begin{cases} x \geq -5 \\ 5 \leq -8 \end{cases} \\ \begin{cases} x < -5 \\ -x-5 \leq x-8 \end{cases} \end{array}\right. \;\Leftrightarrow\; \left[\begin{array}{l} x \in \varnothing \\ \begin{cases} x < -5 \\ 3 \leq 2x \end{cases} \end{array}\right.$$

$$\Leftrightarrow \begin{cases} x < -5 \\ x \geq \frac{3}{2} \end{cases} \;\Leftrightarrow\; x \in \varnothing.$$

Problem 4. Solve the equation:

$$\frac{1}{x} + \frac{1}{x+2} = \frac{x}{x^2 + 2x}.$$

Space for your solution:

We have to find the Least Common Multiple of the three denominators: x, $x+2$ and x^2+2x. Since $x^2+2x = x(x+2)$, the polynomial x^2+2x is the LCM. After we multiply the fractions above by their respective adjoint factors, we will get them all have the same denominator:

$$\tfrac{1}{x} + \tfrac{1}{x+2} = \tfrac{x}{x^2+2x} \;\Leftrightarrow\; \tfrac{1\cdot(x+2)}{x(x+2)} + \tfrac{x\cdot 1}{x(x+2)} = \tfrac{x}{x^2+2x} \;\Leftrightarrow\; \tfrac{(x+2)+x-x}{x(x+2)} = 0 \;\Leftrightarrow\; \tfrac{x+2}{x(x+2)} = 0 \;\Leftrightarrow$$

$$\begin{cases} x+2 = 0 \\ x(x+2) \neq 0 \end{cases} \;\Leftrightarrow\; \begin{cases} x = -2 \\ x \neq 0 \\ x \neq -2 \end{cases} \;\Leftrightarrow\; x \in \varnothing.$$

Problem 5. Perform long division of the polynomial $7x^2-3$ by the polynomial $x+5$. Use the result of the division to write the rational expression $\frac{7x^2-3}{x+5}$ as a sum of a polynomial and another, proper, rational expression.

Space for your solution:

Long division of the polynomial $7x^2-3$ by the polynomial $x+5$ gives:

$$\begin{array}{r|l} & \;\;7x - 35 \\ \hline x+5 & 7x^2 + \;\;0x - 3 \\ & 7x^2 + 35x \\ \hline & \quad -35x - 3 \\ & \quad -35x - 175 \\ \hline & \qquad\quad 172 \end{array}$$

and it means that $\frac{7x^2-3}{x+5} = 7x - 35 + \frac{172}{x+5}$.

Problem 6. Normal saline (NS) is the solution of 9 grams of sodium chloride NaCl (salt) per one liter of water. Since it is similar to human blood in osmolarity, it is used frequently in intravenous drips (IVs), eye drops, nasal washes and to flush wounds and skin abrasions. Sea water contains about 35 grams of salt per liter (the exact figure depends on location). Regular coke contains almost 2 grams of salt per liter.

How much of coke and sea water must be mixed to get 3 liters of a mixture with normal saline concentration?

Space for your solution:

1. Name the unknowns:
   $c =$ (the number of liters of coke to use in the mixture)
   $s =$ (the number of liters of sea water to use in the mixture)

2. Express the known quantities in terms of the unknown ones:
   $$\begin{cases} c+s=3 & \text{— for the volumen of the mixture} \\ \frac{2c+35s}{3}=9 & \text{— for the salt concentration of the mixture} \end{cases}$$

3. Solve the mathematical problem:
   $$\begin{cases} c+s=3 \\ \frac{2c+35s}{3}=9 \end{cases} \Leftrightarrow \begin{cases} c=3-s \\ 2c+35s=9\cdot 3 \end{cases} \Leftrightarrow \begin{cases} c=3-s \\ 2(3-s)+35s=27 \end{cases} \Leftrightarrow$$
   $$\begin{cases} c=3-s \\ 6-2s+35s=27 \end{cases} \Leftrightarrow \begin{cases} c=3-s \\ 33s=21 \end{cases} \Leftrightarrow \begin{cases} s=\frac{21}{33}=\frac{7}{11} \\ c=3-\frac{7}{11}=2\frac{4}{11} \end{cases}$$

4. Translate the solution of the mathematical problem back to the real world:
   We need to use $\frac{7}{11}$ liters of sea water and $2\frac{4}{11}$ liters of coke for our mixture.

Problem 7. Solve the equation

$$3x^2 + 5x - 4 = 0.$$

Space for your solution:

The general quadratic formula (provided that $a \neq 0$) is this:

$$ax^2 + bx + c = 0 \quad \Leftrightarrow \quad x = \tfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

which gives us for our equation:

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3} = \frac{-5 \pm \sqrt{25 + 48}}{6} = \frac{-5 \pm \sqrt{73}}{6}.$$