Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

Wednesday, May 13, 2026

MAT 120
College Algebra and Trigonometry

Final Exam: Solutions and Answers

Instructor:

Name: Alexander Kasiukov
Office: Suffolk Federal Credit Union Arena, Room A-109
Phone: (631) 851-6484
Email: kasiuka@sunysuffolk.edu
Web Site: http://kasiukov.com

Problem 1. In this problem, we will consider functions $3 + \log_2 x$ and $\log_2(3 + x)$.

(1). Solve the equation $3 + \log_2 x = \log_2(3 + x)$ analytically.

Space for your solution:

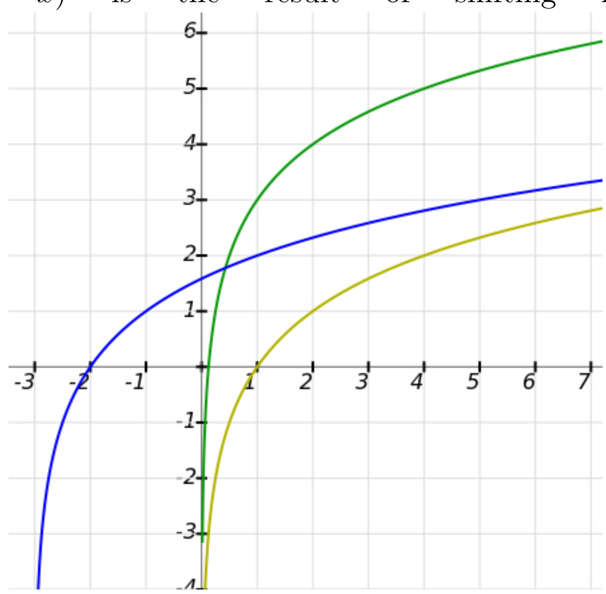
$$3 + \log_2 x = \log_2(3+x) \quad \Leftrightarrow \quad 3 = \log_2(3+x) - \log_2 x \quad \Leftrightarrow \quad \begin{cases} 3 = \log_2 \frac{3+x}{x} \\ 3 + x > 0 \\ x > 0 \end{cases} \quad \Leftrightarrow$$

$$\begin{cases} 2^3 = 2^{\log_2 \frac{3+x}{x}} \\ x > -3 \\ x > 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} 8 = \frac{3+x}{x} \\ x > 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} 8x = 3 + x \\ x > 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} 7x = 3 \\ x > 0 \end{cases} \quad \Leftrightarrow \quad x = \tfrac{3}{7}.$$

(2). Using the technique of graph transformations, sketch the graphs of these functions in the same (x, y)-coordinate system. Is this sketch consistent with your solution of part (1)?

Space for your solution:

The graph of $3 + \log_2 x$ is the result of shifting $\log_2 x$ up by 3, and $\log_2(3 + x)$ is the result of shifting $\log_2 x$ left by 3:



$\log_2(3 + x)$, $\log_2 x$, $3 + \log_2 x$.

These two resulting grapsh do intersect at the point consistent with the previous solution $x = \frac{3}{7}$.

Problem 2. Solve the equation $3^{1-2x} = 4^x$.

Space for your solution:

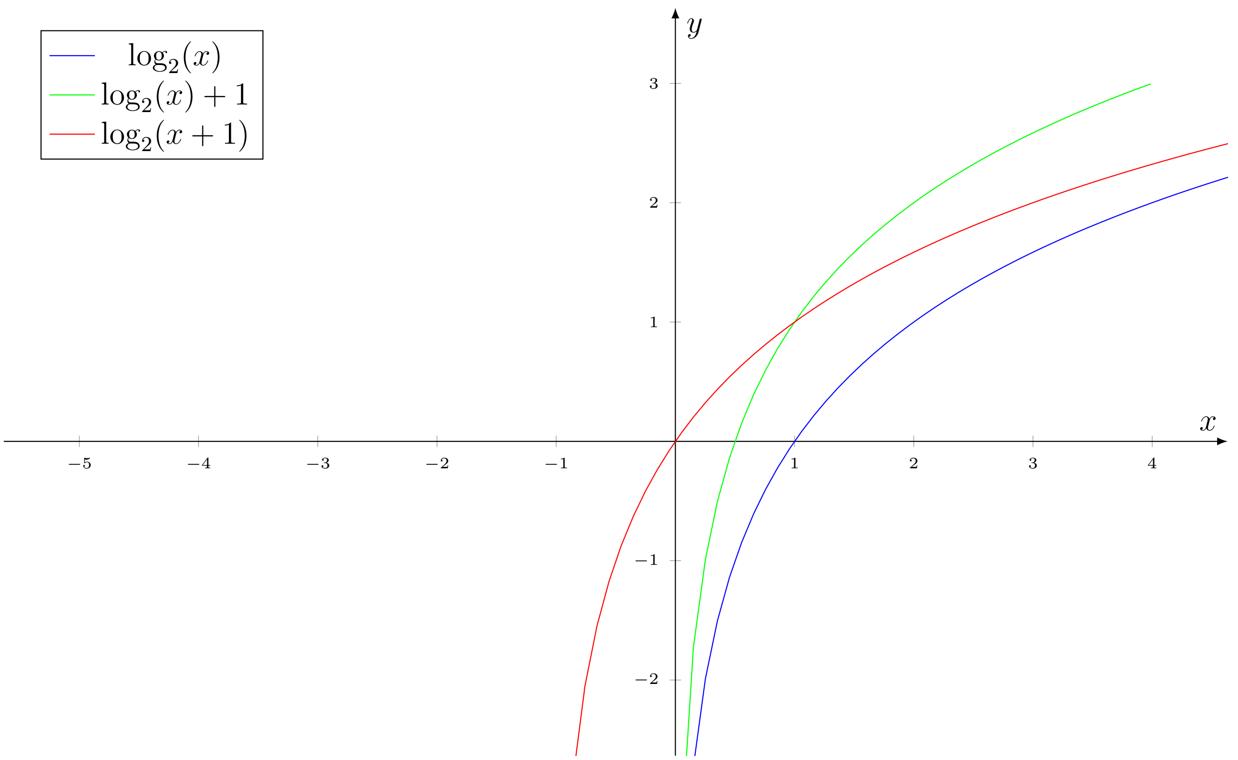
$$3^{1-2x} = 4^x \quad \Leftrightarrow \quad \ln 3^{1-2x} = \ln 4^x \quad \Leftrightarrow \quad (1 - 2x)\ln 3 = x \ln 4 \quad \Leftrightarrow \quad (\ln 3) - x \cdot (2 \ln 3) = x \ln 4$$

$$\Leftrightarrow \quad x = \frac{\ln 3}{\ln 4 + 2\ln 3}.$$

Problem 3. Use transformations of the graph $\log_2(x)$ to sketch, in the same system of coordinates, the graphs of the two functions, $\log_2(x) + 1$ and $\log_2(x + 1)$.

Space for your solution:

The graph of $\log_2(x)+1$ is the result of shifting that of $\log_2(x)$ up by 1, whereas $\log_2(x+1)$ is obtained by shifting the same graph left by 1.



Problem 4. Solve the equation

$$\log_2(x) + 1 = \log_2(x + 1)$$

and check if your solution is consistent with the above sketch.

Space for your solution:

$$\log_2(x) + 1 = \log_2(x + 1) \Leftrightarrow 2^{\log_2(x)+1} = 2^{\log_2(x+1)} \Leftrightarrow 2^{\log_2(x)} \cdot 2^1 = 2^{\log_2(x+1)} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x \cdot 2 = x + 1 \\ x > 0 \\ x + 1 > 0 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ x > 0 \\ x + 1 > 0 \end{cases} \Leftrightarrow x = 1.$$

The two resulting graphs in the previous sub-problem intersect in a single point, confirming that the equation in question has a unique solution.

Problem 5. Consider the function with the range $\mathbb{R}$, defined by the formula

$$f(x) = \frac{2x^3 - 5x^2 + 7}{x^2 - 4x + 4}$$

for all $x \in \mathbb{R}$, for which the above formula makes sense.

(1). What is the domain of the function f?

Space for your solution:

The above formula makes sense if and only if the denominator $x^2 - 4x + 4$ of the fraction defining $f(x)$ is not zero. Since $x^2 - 4x + 4 = 0 \quad \Leftrightarrow \quad (x-2)^2 = 0 \quad \Leftrightarrow \quad x = 2$, we get:

$$\texttt{Dom}\ f = \{x \in \mathbb{R} : \ x \neq 2\}.$$

(2). Find all the vertical asymptotes of the graph of $f(x)$.

Space for your solution:

The graph of $f(x)$ has vertical asymptotes when the denominator of the fraction turns into zero (while the numerator stays non-zero). The denominator $x^2 - 4x + 4 = (x-2)^2$ has a single root $x = 2$ of multiplicity 2. Thus the line $x = 2$ is the only vertical asymptote of $f(x)$.

(3). Find the y-intercept of $f(x)$.

Space for your solution:

The y-intercept is the value of the function that corresponds to the input $x = 0$, thus the y-intercept is $f(0) = \frac{7}{4}$.

(4). Perform long division of the numerator of $f(x)$ by its denominator. Using the results of the long division, write $f(x)$ as a sum of a polynomial and a proper fraction.

Space for your solution:

$$\begin{array}{r|l} & \quad\quad\quad\quad 2x + 3 \\ \hline x^2 - 4x + 4 & \ \ 2x^3 - 5x^2 + 0x + 7 \\ & -(2x^3 - 8x^2 + 8x) \\ \hline & \quad\quad 3x^2 - 8x + 7 \\ & \quad -(3x^2 - 12x + 12) \\ \hline & \quad\quad\quad\quad 4x - 5 \end{array}$$

The above long division means that

$$f(x) = 2x + 3 + \frac{4x - 5}{x^2 - 4x + 4}.$$

(5). Find the equation of the oblique asymptote of $f(x)$.

Space for your solution:

The oblique asymptote of $f(x)$ is the line $y = 2x + 3$.

(6). Find all the intersections of the graph of $f(x)$ with the oblique asymptote. (Only the x-coordinates of the intersections are needed.)

Space for your solution:

The graph of $f(x)$ will intersect the oblique asymptote whenever the numerator of $\frac{4x-5}{x^2-4x+4}$ turns into zero. The numerator has a single root $x = \frac{5}{4}$ of multiplicity 1.

(7). Use the Rational Roots Theorem to find a rational root of $2x^3 - 5x^2 + 7$.

Space for your solution:

The Rational Roots Theorem states that any rational root of this polynomial will be in the set:

$$\left\{\frac{\text{divisor of } 7}{\text{divisor of } 2}\right\} = \left\{1, -1, 7, -7, \frac{1}{2}, -\frac{1}{2}, \frac{7}{2}, -\frac{7}{2}\right\}.$$

Trying those values of x in $2x^3 - 5x^2 + 7$ yields the root $x = -1$.

(8). Use the result of the previous subproblem to find all x-intercepts of the funciton $f(x)$.

Space for your solution:

The x-intercepts of the function $f(x)$ are the roots of its numerator $2x^3 - 5x^2 + 7$. In the previous subproblem, we found one of its roots $x = -1$. The Polynomial Remainder Theorem of Bézout guarantees divisibility of $2x^3 - 5x^2 + 7$ by $x + 1$:

$$\begin{array}{r|l} & 2x^2 - 7x + 7 \\ x+1 & 2x^3 - 5x^2 + 0x + 7 \\ & -\ (2x^3 + 2x^2) \\ & \quad -7x^2 + 0x \\ & \quad -\ (-7x^2 - 7x) \\ & \qquad 7x + 7 \\ & \qquad -\ (7x + 7) \\ & \qquad\quad 0 \end{array}$$

Thus $2x^3 - 5x^2 + 7 = 0 \quad \Leftrightarrow \quad (x+1)(2x^2 - 7x + 7) = 0 \quad \Leftrightarrow \quad \left[\begin{array}{l} x + 1 = 0 \\ 2x^2 - 7x + 7 = 0 \end{array} \right.$

The quadratic equation in the above system has no solution because its discriminant $D = b^2 - 4ac = (-7)^2 - 4 \cdot 2 \cdot 7 = 49 - 56 = -7$ is negative. Therefore $x = -1$ is the only x-intercept of the function $f(x)$.

(9). Use the result of the previous sub-problems to sketch the graph of the function f. Mark all vertical, horizontal and oblique asymptotes, as well as all intersections of the graph with the asymptotes and the axis, if any.

Space for your solution: