Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

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MAT 141
Calculus with Analytic Geometry I

Final Exam

Instructor:

Name: Alexander Kasiukov

Office: Suffolk Federal Credit Union Arena, Room A-109

Phone: (631) 851-6484

Email: kasiuka@sunysuffolk.edu

Web Site: http://www.kasiukov.com

Please print the requested information in the spaces provided:

Student:

Name:

Student Id:

Email: *include to receive the final grade via email* ONLY *if you are not getting email updates*

- *Notes and books are permitted on this exam.*
- *Graphing calculators, smartwatches, computers, cell phones and any other communication-capable devices are prohibited. Their mere presence in the open (even without use) is a sufficient reason for an immediate dismissal from this exam with a failing grade.*
- *You will not receive full credit if there is no work shown, even if you have the right answer. Please don't attach additional pieces of paper: if you run out of space, please ask for another blank final.*

Problem 1. Consider the following claim:

$$\lim_{x\to 0+} \Big(\ln(x)\Big) = -\infty.$$

(1). Make a sketch of the graph of the function ln and determine, based on the graph, if the above claim is true.

Space for your solution:

(2). Using the definition of limit, translate the above claim into a statement about inequalities.

Space for your solution:

(3). Based on the definition of limit, determine if the claim in question is true. (Call M the variable that controls closeness of $\ln(x)$ to $-\infty$ and δ — the one that controls the closeness of x to 0. Demonstrate the truth of the limit-claiming statement by providing an explicit value of δ for any given M, or the falsehood — by producing a counterexample value of M.)

Space for your solution:

Problem 2. Use the limit of composition theorem and cancellation of the main terms (based on the properties of logarithms) to find

$$\lim_{x \to 0+} \left(\ln\left(10x^2\right) - \ln\left(3x\right) \right).$$

Space for your solution:

Problem 3. Consider the function with range $\mathbb{R}$ defined as $f(x) = \ln(x)$ on the maximum set of real numbers x for which this formula makes sense.

(1). What is the domain of the function f?

Space for your solution:

(2). Is the function $f(x)$ continuous at $x = 5$? Why? (No proof is needed, but please state the meaning of continuity in terms of limits.)

Space for your solution:

(3). Is the function f continuous?

Space for your solution:

(4). Suppose that

- the domain $(0, +\infty)$ of the function f is compactified by adding 0 and $+\infty$, and
- the range $\mathbb{R}$ of f is compactified by adding $\{-\infty, +\infty\}$.

Can the function f be extended to become a continuous function from $[0, +\infty]$ to $\mathbb{R} \cup \{-\infty, +\infty\}$? If so, provide the extention; if not, give the reason why it is impossible.

Space for your solution:

Problem 4. Find $\frac{\mathtt{d}\sin\left(\frac{x^2}{\cos(x)}\right)}{\mathtt{d}x}$. (The answer should not contain any operations of derivative, but other than that does not have to be simplified in any way.)

Space for your solution:

Problem 5. In this problem we will study the extrema and the points of extrema of the function $f(x) = 4x - 3x^3$ on the interval $[0, 2]$.

(1). Identify the suspect points of extrema.

Space for your solution:

(2). Find the points of extrema and the extrema of the function f.

Space for your solution:

Problem 6. The arresting gear system of an aircraft carrier enables landing of high-speed aircraft within limited distance. It includes an arresting wire that is layed across the deck. The wire is grasped and pulled out by the aircraft's tailhook. The arresting wire transmits tension to hydraulic arresting gears, which dissipate the kinetic energy of the aircraft by hydraulic damping.

In this problem we will derive the Young's modulus, i.e. the damping ratio, of material that is needed to stop an aircraft landing on a deck of an aircraft carrier, in a given space. We will use a very simplistic model of hydraulic damping, assuming that the damping mechanism works as an elastic spring with force $F = H \cdot x$, where F is the damping force (in Newtons), x is the displacement (in meters), and H is the Young's modulus (in $\frac{kg}{m \cdot s^2}$).

(1). Find the formula, in terms of H and d, for the work $W = \int_0^d F(x)\, dx$ against the damping force F done when moving an aircraft along the deck for d meters from the relaxed position of the arrester wire.

Space for your solution:

(2). As the aircraft moves along the deck, it uses up its kinetic energy $E = \frac{M \cdot v^2}{2}$ to do the work against the dumping force. In order for the aircraft's speed to be reduced to zero at the end of landing, the initial kinetic energy E of the aircraft must equal to the total amount of work W done by the arresting gear system. Using the formula for W from above, express the Young's modulus H sufficient for stopping an aircraft of mass M with the landing speed v over the landing deck of length d.

Space for your solution:

(3). Assume that the mass of the F-18 Hornet at landing is $18,000$ kg, its landing speed is 60 $\frac{\text{m}}{\text{s}}$, and the landing runway is 200 m long. Compute the Young's modulus sufficient for these conditions.

Space for your solution: